The History of Logic

Aristotle (384–322 BC) invented logic.

- Predecessors: Fred Flintstone, geometry, sophists, pre-Socratic philosophers, Socrates & Plato.
- Syllogistic logic, laws of non-contradiction and excluded middle, modal logic.
- Some logic in a "wider sense": definitions, fallacies, inductive reasoning, etc.
- Parallels in "Buddhist logic."
- The basis for the "traditional logic" that dominated until the early 20th century.

The Stoics developed a rival logic.

- A propositional logic, focusing on "and," "or," and "if-then" (controversies).
- Much about modal logic and determinism.
- Philosophy had three parts: physics, ethics, and logic.
- Eventually merged with the "traditional logic" based on Aristotle.

The Medievals developed logic further.

- Boethius: modal logic and translations of Aristotle.
- Arab logic and rediscovery of Aristotle's writings.
- Refined the details: Barbara-Celarent, rules for valid syllogisms, university textbooks.
- Modal logic, problem of universals, etc.
- Much use of logic in philosophy (e.g. Thomas Aquinas); we still use many Latin terms in logic (like "modus ponens" and "a priori").

Renaissance to 19th century logic

- The Enlightenment (e.g. Kant) brought little progress in logic.
- Leibniz proposed a symbolic logic that would reduce reasoning to a kind of calculation.
- Hegel and Marx proposed a *dialectical logic*.
- Others (like De Morgan) proposed new ways to symbolize logical operations.

George Boole (1815–64) invented "mathematical logic."

- Letters stand for sets. So "H" might stand for the set of humans" and "HM" might stand for the set of entities that are both humans and mortals. Then "all humans are mortals" is "H = HM."
- We can symbolize a syllogism as a series of equations and validate it algebraically:

All humans are mortal.	$\mathbf{H} = \mathbf{H}\mathbf{M}$
All Greeks are humans.	G = GH
All Greeks are mortal.	\therefore G = GM

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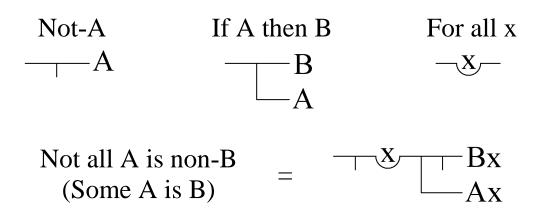
• "Boolean algebra" can be interpreted to be about sets or about statements. For example:

-A	The set of non-As	Not-A
A∩B	The intersection of sets A and B	A and B
A∪B	The union of sets A and B	A or B

- Similar laws cover both interpretations, like " $A \cap B = B \cap A$ " and "- $(A \cap B) = (-A \cup -B)$."
- Boole wanted mathematicians to take over logic. But logic today is studied in philosophy, mathematics, and computer science departments.

Gottlob Frege (1848–1925) invented "classical symbolic logic."

- This overcame the gap between Aristotelian and Stoic logic in a higher synthesis.
- Frege used lines for "not," "if-then," and "all":



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- Frege tried to show that arithmetic was reducible to logic: every arithmetic truth can be formulated using just notions of logic and proved using just axioms and inference rules of logic.
- One of Frege's axioms said that every condition on x (like "x is a cat") picks out a set. But then "x is not a member of itself" (x ∉ x) picks out a set R containing just those things that are not members of themselves. Then:

For all $x, x \in R$ if and only if $x \notin x$. Therefore, $R \in R$ if and only if $R \notin R$.

Bertrand Russell (1872–1970) tried to put classical logic on a firmer basis.

- *Principia Mathematica* (1910–1913, co-authored with Alfred North Whitehead) used a better notation and tried to avoid "Russell's Paradox."
- This became the standard formulation of *classical symbolic logic* which gradually replaced *traditional logic* to became the new orthodoxy.

After Principia

- Wittgenstein created truth tables, which gave a *semantic* test of validity.
- Principia's systematization of propositional and quantificational logic was later shown to be sound and complete.
- Kurt Gödel in 1931 showed, against Frege and Russell, that arithmetic was not reducible to any sound and complete axiomatic system.
- Classical logic played a big role in the birth of modern computers.

Non-classical logics arose.

- Supplementary non-classical logics include modal, deontic, belief, and temporal logics.
- *Deviant non-classical logics* include multi-valued and paraconsistent logic.

Logic "in a broad sense" is much pursued.

- *Informal logic* covers various non-formal skills that we need to appraise reasoning.
- *Inductive logic* is about forms of reasoning in which we extrapolate from observed patterns to conclude that a given conclusion is probably true.
- *Metalogic* is the study of formal systems.
- *Philosophy of logic* deals with wider philosophical issues raised by logic (like "What is truth?" and "Are there abstract entities?").