Chapter 12

Frontiers of Logic—Fuzzy Logic: Can Aristotle and the Buddha get along?

Fuzzy logic begins where Western logic ends . . . . Fuzziness begins where contradictions begin, where A and not-A holds to any degree.

--- Bart Kosko, ***Fuzzy Thinking: The New Science of Fuzzy Logic***

Everything must either be or not be, whether in the present or in the future.

--- Aristotle, ***On Interpretation***

I have not explained that the world is eternal or not eternal, I have not explained that the world is finite or infinite.

--- The Buddha

The fundamental idea of Buddhism is to pass beyond the world of opposites, a world built up by intellectual distinctions and emotional defilements.

--- D.T. Suzuki, ***The Essence of Buddhism***

Introduction

I have a complaint about the college where I teach. Complaining to my administration will not do any good, for the source of my complaint appears to be at a very deep philosophical level beyond any administrator's control. I teach in Hawaii. Outside right now it is about 85 degrees, but as I type these words in my office I have on a thick jacket. The temperature in my office and adjacent classrooms averages between 55 and 60 degrees. I used to be able to control the temperature in my office and my classroom by adjusting the old analog thermostats in each room. True, there was some minor inconvenience in the process, and at times energy was wasted when someone turned a thermostat down and then forgot to turn it back up when leaving for the day or a meeting in another building. On Monday mornings I would have to turn the thermostat way down to cool my office, which had no air conditioning all weekend, and then turn it back up when it eventually got too cold. I would also have to adjust my classroom thermostat depending on the time of day and how many people were on my floor of the building and in the classroom.

But with a little effort I could get the temperature right, because as a human being I could assess and "smoothly" control the temperature.

Now we have a so-called state-of-the-art digital computerized system, intended to centrally control each room in each building and eliminate inconvenience and save taxpayers lots of money. So now we all freeze and wear jackets in Hawaii, and waste lots of energy. We don't dare complain. If our comptroller adjusts the system it will be too hot, and for a teacher, having an office and a classroom that are too hot is a fate worse than death. The mind shuts down, and suddenly even your best lecture becomes boring and students begin to fall asleep.

According to the gurus of a new logic, called ***fuzzy logic***, the root of our problem is cultural and philosophical: Our air conditioning system thinks like Aristotle rather than like the Buddha. According to the proponents of this new logic, the all-or-nothing overshoot of our air conditioning system is the technological end-product of a cultural hasty conclusion fallacy in regard to truth. Since the time of Aristotle and the ancient Greeks, Western logic has assumed that a proposition or statement must be wholly true or false with no in-between and no shades of gray.[1] Small wonder that our computer systems are dumb, proponents of fuzzy logic say, if they are programmed on the basis of a black and white logic. Based on such notions of categorical truth and falsehood, on-and-off systems have no common sense; they are incapable of mimicking the simple human process of smoothly adjusting a thermostat when a room is too hot or too cold.

Perhaps it is misleading to title this chapter "frontiers" of logic. For proponents of this new logic claim that a fuzzy revolution has taken place for some time now in Buddhist-influenced countries such as Japan, Singapore, Malaysia, South Korea, Taiwan, and China, with billions of dollars invested in fuzzy controlled cameras, camcorders, TV's, microwave ovens, washing machines, vacuum sweepers, car transmissions, and engines. Now we have very quiet washing machines able to measure the amount of dirt in wash water and smoothly control the amount and length of water agitation to get clothes clean, fuzzy vacuum cleaners controlled by fuzzy decision rules that imperceptibly adjust sucking power in microseconds based on sensor readings of dirt density and carpet texture, a Neuro Fuzzy® Rice Cooker “that 'thinks' for itself,” rice cookers with special sensors to observe the rice as it cooks, adjusting the cooking for the type of rice and volume, changing the temperature when necessary, and TV sets that instantly measure each picture frame for brightness, contrast, and color, and then smoothly adjust each, microsecond by microsecond, like a thousand little nano-creatures turning knobs with common sense.[2] In addition, say proponents, soon we will be able to create truly intelligent computers, ***adaptive fuzzy systems***, that will think much more like humans than current computers, computers able to learn, see patterns and "grow" rules based on these patterns. All of this is based on what the Japanese call "fuaji riron" -- fuzzy theory.

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[1] In Chapter 2, Plato was blamed for the notion of categorical truth. Aristotle was a student of Plato. Aristotle is the target of proponents of fuzzy logic because Aristotle was the first to systematize many aspects of Western logic based on the generalization that statements are either wholly true or false.

[2] *Nano* means a *billionth of* as in a billionth of a second or a billionth of a meter.

Bivalent Logic and Paradoxes

According to the proponents of fuzzy logic, we did not have to wait for faulty air conditioning systems to know that something was seriously wrong with the foundations of Western logic and our assumptions regarding truth. Classical Aristotelian logic is said to be founded on a ***bivalent*** faith, propositions (statements) are "crisply" true or false. But our experience tells us there are many areas of life where a crisp categorical bivalent map oversimplifies to the point of paradox by missing important shades of gray. In short, by not recognizing that there are degrees of truth between the extremes of complete truth and complete falsehood, there is a mismatch problem between our logic and reason on the one hand, and our experience on the other hand. For instance, using Western logic founded on bivalent faith it is possible to prove the following arguments to be valid:

1. If 100,000 grains of sand make a heap of sand, and removing one grain of sand still leaves a heap, it follows that one grain of sand is still a heap.

2. If an object x is black, then any object y that is indistinguishable in color from x is also black. Consequently, any white object z is black that was produced by a sequence of gray shading such that the percentage of white increased smoothly such that each step in color was indistinguishable from the previous one.

3. If a person who is only 5 feet tall is short, then given a sequence of 999 additional persons such that starting with our 5-foot-tall person, one by one is 1/32 of an inch taller than the previous person, the last person in the sequence, a little over 7 feet 6 inches tall, is also short.

These paradoxes are known as ***Sorites paradoxes*** or ***paradoxes of vagueness***. Let's examine the last case in more detail. Suppose I was able to find a thousand people, such that the shortest was only five feet tall and the tallest was a little over seven feet, six inches. Starting with our 5-foot-tall person, I was able to find a person who was 5 feet 1/32 of an inch tall, then a person 5 feet 2/32ths of an inch tall, then 5 feet 3/32ths of an inch, and so on. Clearly, if the person who is only 5 feet tall is short, then the person who is 5 feet 1/32 of an inch is also short. But given this, we can now create a long series of valid modus ponens' steps[3] as follows:

x_1 is short. ($x_1 = 5'$)

[3] Or a long series of hypothetical syllogisms and one modus ponens step. This paradox is called a sorites paradox because it can be viewed as a long series of arguments where the conclusion of each becomes a premise for the next argument.

If x_1 is short, then x_2 is short. (x_2 = 5' 1/32")
If x_2 is short, then x_3 is short. (x_3 = 5' 2/32")
:
:
:
If x_{999} is short, then x_{1000} is short.

/∴ x_{1000} is short. ($x_{1000} \approx$ 7' 6")

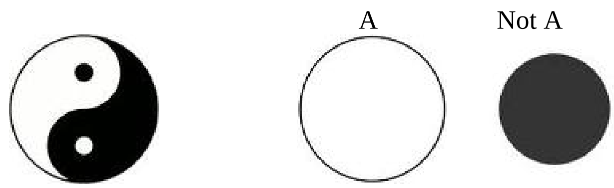
Our common sense rebels against this conclusion. The last person in the sequence (x_{1000}) is a little over seven feet six inches tall and is not short, but there is nothing wrong with the logic—if by logic we mean Western logic and bivalent truth values. Modus ponens is correctly applied, and given the traditional interpretation of a conditional statement, none of the premises are false. There is no conditional in the sequence that has a true antecedent and a false consequent. If any given person mentioned in the antecedent is short, then a person mentioned in the consequent must be short as well, because the person mentioned in the consequent is virtually indistinguishable from the person mentioned in the antecedent. Remember that the person mentioned in the consequent is only 1/32 of an inch taller than the person mentioned in the antecedent. So how could it be that a person mentioned in the antecedent is short and a person mentioned in the consequent is not short? Although there is a difference in height between the persons mentioned in the antecedent and the consequent, it is too small to provide a reason to apply *short* to one person and withhold it from the other.

The problem with this logical picture is that *short* is a fuzzy concept. Membership in the class of short human beings does not have a crisp cutoff point, such that one person in our sequence is wholly short and the next in the sequence wholly not short. However, our judgment of the truth value of conditional statements demands crisp true or false antecedents and consequents. A conditional statement is false only when the antecedent is wholly true, and the consequent is wholly false.

Recall that the use of Venn diagrams covered in the previous chapter assumed a crisp division between classes of things. In Venn diagrams, the lines that mark off each class of things are sharp. Although classes can overlap—something could be a class A and a class B at the same time—something cannot be both in an A class of things and not in an A class of things at the same time. In modern logic, classes of things are called ***sets*** and fuzzy logic is based on the belief that classical set theory is wrong; our experience and the categorization of our experience is based on ***fuzzy sets***. There are no sharp lines, as in Venn diagrams; one fuzzy set of things can blend into another fuzzy set of things.

Consider our previous discussion of our ability at a very young age to grasp abstract concepts. Plato and Aristotle assumed that a crisp in-principle definition or conceptual apprehension of an ideal chair or chairness was possible. Fuzzy theorists reject this and argue that inclusion in a set is a matter of degree. No sharp definition is possible, because some objects may be definitely chairs, but others are "sort of" chairs. Whereas Venn diagrams picture the categorical world view of Western logic with its crisp separation of classes, the Tao yin-yang symbol of Eastern philosophy reflects the blending, shades of gray world view of fuzzy logic. (Figure 12-1) In fuzzy theory, truth is a matter of accuracy, and accuracy must be measured in degrees.

Figure 12-1
Whereas crisp Venn circles (right) picture the categorical, either/or world of Aristotelian logic, the Tao yin-yang symbol of Eastern philosophy reflects the blending, shades-of-gray world view of fuzzy logic.



Ironically Western logicians and philosophers discovered the above paradoxes in attempting to completely systematize Western rationality, that is, logic and mathematics. The British philosopher and logician Bertrand Russell (1872-1970) once asked what we should make of a barber in need of a shave who posts a sign outside his shop that reads, "I shave all, and only, those men who do not shave themselves." Who would shave the barber? If he shaves himself, then according to his sign he does not; if he does not shave himself, then according to the sign he does. Logically a contradiction follows from this:

(S = The barber shaves himself)

1. $(S \supset {\sim}S) \bullet ({\sim}S \supset S)$ / $\therefore$ $S \bullet {\sim}S$	
2. $S \supset {\sim}S$	(1) Simp.
3. ${\sim}{\sim}S \supset {\sim}S$	(2) DN
4. ${\sim}S \vee {\sim}S$	(3) Impl.
5. ${\sim}S$	(4) Rep.
6. ${\sim}S \supset S$	(1) Com. + Simp.
7. $S \vee S$	(6) Impl.
8. S	(7) Rep.
9. $S \bullet {\sim}S$	(8)(5) Conj.

But a contradiction is a disaster for Western logic, because in addition to being necessarily wholly false and hence, making any argument that contains a contradiction in the premises unsound, any conclusion whatsoever can be derived from a contradiction.

1. $p \bullet {\sim}p$ / $\therefore$ q	
2. p	(1) Simp.

3. p v q	(2) Add.
4. ~p	(1) Com. + Simp.
5. q	(3)(4) D.S.

Similar paradoxes are: The man from Crete who says "All Cretans are liars"; the statement, "This statement is false"; and a bumper sticker that says, "Don't trust me." For the defenders of fuzzy logic, these paradoxes are not merely ivory-tower curiosities. They are nature's way of telling us that we have not got something right, that there is something fundamentally wrong—or incomplete to be more precise—with our Western world-view. According to the proponents of fuzzy logic, the solution to this mismatch between reason and experience, and to our overshooting air-conditioning systems, is to see ***Western logic as only a special case of an expanded logic***, a more general rationality that allows for, in fact demands, degrees of truth.

Multivalued Logic and Degrees of Truth

Thus, the fundamental move of fuzzy logic, or more appropriately called **multivalued logic**, is to generalize from classical logic by having the bivalent truth values of completely true and completely false be the extreme endpoints in a continuum of degrees of truth. If a statement is wholly true, a 1 is assigned; if a statement is wholly false, a 0 is assigned. We can then speak of a domain of discourse (short people, for instance) and a fuzzy *set* in that domain (the grouping of all short people), and the degree to which a particular person or object is in the set. Here are some typical fuzzy examples:

Person	Height	Degree of shortness
Midge	5' 0"	1.00
Bea	5' 1/32"	0.99
Hyon	5' 6"	0.80
Taisha	6' 0"	0.60
Aron	6' 6"	0.40
Kuna	7' 0"	0.20
Jorge	7' 5"	0.03
Kareem	7' 6"	0.00

Thus, statements such as "A is short" can now be interpreted in terms of degrees of truth, as the degree to which A participates in the fuzzy set *short*. For instance, the statement "Aron is short" is .40 true.[4]

[4] Although there is no one assignment of degrees that can be said to be absolutely correct, fuzzy logicians talk about a "reasonable assignment of degrees," given two extremes. A typical method is to assign 0 to the lowest value and 1 to the highest value, then any intermediate value will equal the original quantitative assignment (in this case height) minus the lowest value divided by the difference between the lowest and highest values. The above intermediate degrees were

The next move in fuzzy logic is to redefine the propositional logical connectives (**~**), (**•**), and (v**)**. The guideline to be used in this redefinition is called the ***Extension Principle***. The traditional Western truth values for these logical connectives are "recovered" when they connect simple statements that have the crisp values of 1 or 0, when simple statements are wholly true or wholly false. In other words, **fuzzy logic is not incompatible with traditional logic, but rather a generalization from traditional logic. Western logic is not wrong; it is simply incomplete and needs to be extended**. With some help from the Buddha and Eastern philosophy, the conception of rationality is enlarged. Fuzzy logic does not endorse mysticism and irrationality. Fuzzy logic provides increased precision with a new set of rational guidelines that allows dealing in a practical way with aspects of our experience that bivalent logic seems incapable of capturing. The rules of fuzzy logic themselves are not fuzzy or vague. Here are the rules:

Negation (~): The degree of truth of **not A** = 1.0 **–** (minus) the degree of truth of A.

Example: "A is short" = .40 (so)
"A is **not** short" = .60 (1-.40) = .60

Conjunction (•): The degree of truth of **A•B** = the **minimum** degree of truth of **A** and **B**.

Example: (.40) • (.99) = .40

Disjunction (v): The degree of truth of **AvB** = the **maximum** degree of truth of **A** and **B**.

Example: (.40) v (.90) = .90

Consider how each of these rules is a simple extension of the traditional truth table definitions. A negation returned the opposite value of that negated (~T = F; ~1 = 0; so ~(.40) = (1-.40) = .60). A traditional conjunction returned the minimum value of the parts conjoined (T•F = F; 1•0 = 0; so .40 • .99 = .40). A traditional disjunction returned the maximum value of its parts (TvF = T; 1v0 = 1; so .40 v .99 = .99). Consider now what a fuzzy truth table would look like.

If p = X is ***not*** short.
q = X is short ***and*** X is young.
r = X is short ***or*** X is young.

then we can compute the following values:

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achieved in the following way. Given that 5' equals 1 and 7' 6" equals 0, then any intermediate height ***x*** equals 1 - [(x-5)/2.6]. For Aron, (in inches) 1 – [(78-60/30] = 1-(18/30) = 1- .60 = .40.

Height	Age	X is short	X is young	p	q	r
5' 0"	10	1.0	1.0	0.0	1.0	1.0
5' 1/32"	12	0.99	0.96	.01	.96	.99
5' 6"	65	0.80	0.00	.20	0.0	.80
6' 0"	55	0.60	0.18	.40	.18	.60
6' 6"	18	0.40	0.85	.60	.40	.85
7' 0"	25	0.20	0.73	.80	.20	.73
7' 5"	45	0.03	0.36	.97	.03	.36
7' 6"	29	0.00	0.65	1.0	0.0	.65

Proponents of fuzzy logic claim that the assignment of partial truth plus these new definitions of logical connectives provide us with a power of discrimination (and hence, precision) lacking in classical logic. It provides a method for handling shades of gray that is more consistent with common sense. A person who is only 5 feet 1/32 of an inch tall and only 12 years old is very close to a person only 5 feet tall and only 10 years old (.96 and 1.00 respectively), but very different than a person who is 7 feet 5 inches tall and 45 years of age (.96 and .03 respectively).

Fuzzy Conditionals and Fuzzy Validity

In classical logic there is an important relationship between valid conclusion and conditional statements that are tautologies. For instance, a truth table of the conditional, **[p • (p ⊃ q)] ⊃ q** will show a result of all true in its final column. Note that this conditional is formed by conjoining the premises of a modus ponens argument form and making them the antecedent of a conditional with the form's conclusion as the consequent.

p q	[p • (p ⊃ q)] ⊃ q		*
T T	T	T	**T**
T F	F	F	**T**
F T	F	T	**T**
F F	F	T	**T**

p ⊃ q
p /∴ q

All of the nine rules of inference can be rendered this way, and all will produce tautologies: for modus tollens, **[(p ⊃ q) • ~q] ⊃ ~p**, for disjunctive syllogism, **[(p v q) • ~p] ⊃ q**, and so on. Similarly, a truth table for the rules of replacement will show each to be a tautology. As we saw in Chapter 10, a truth table for the rule of implication, (~p ⊃ q) ≡ (p v q), will show all true under the (≡) symbol.

From a mutivalued logic perspective these relationships reflect the definition for a crisp notion of validity: That a valid argument not allow for any possibility where the conclusion is false and the premises are all true. This is why a valid rule of inference reformulated into a statement will always be a tautology: there will be no row in a truth table with a true antecedent and a false consequent. As in the case of the logical connectives, in fuzzy logic the notion of validity is also generalized: ***a completely* (100%) *fuzzy valid argument is one that does not allow for a loss of truth in going from the premises to the conclusion***. For example, an argument is not completely fuzzy valid if its premises have an overall degree of truth of .5 and the conclusion has a value of .4.

Similarly, the notion of a classical conditional statement is generalized. One way of viewing the traditional *if . . . then* statement defined in Chapter 8 is: A conditional statement is wholly true when its antecedent is no more true than its consequent, otherwise it is wholly false. In other words, when the antecedent is equal to the consequent in truth value, T ⊃ T or F ⊃ F, or has less truth value, F ⊃ T, an if-then statement is wholly true. Only when there is a loss of truth in moving from the antecedent to the consequent, T ⊃ F, is an *if . . . then* statement wholly false. In fuzzy logic the first aspect is maintained. If there is no loss of truth, then a conditional statement is wholly true. However, if there is only a small loss of truth in moving from the antecedent to the consequent, such as in the statement "if .5 then .4," we want to distinguish this case from one where there is a large loss of truth, such as in the statement "if .99 then .03." A classical *if . . . then* definition loses the ability to make this discrimination by allowing only one choice, complete falsehood. Thus, <u>the idea is to define a fuzzy conditional such that the degree of truth reflects how much truth is lost in the passage from the antecedent to the consequent</u>. Here is the rule for fuzzy *if . . . then* statements, followed by that for biconditionals. (To reflect the important difference between the classical definitions and the fuzzy definitions, let's use the (→) symbol rather than the (⊃) symbol, and (↔) instead of (≡).)

Conditional (→): The degree of truth of **A → B** = 1-(A-B) if **A** is greater than **B**, otherwise 1.

Examples:

.5 → .4= 1-(.5-.4)	= 1-.1	= .9	
.99 → .03	= 1-(.99-.03)	= 1-.96= .04	
1.0 → 0.0	= 1-(1-0)	= 1-1	= 0
0.0 → 1.0			= 1
.5 → .6		= 1	

Biconditional (↔): The degree of truth of **A ↔ B** = (A→B)•(B→A).[5]

[5] This, of course, is a derived definition showing that one version of the Equivalence rule of replacement works for fuzzy logic. The definition often found in the literature on fuzzy logic is **A ↔ B** = 1-│A-B│. │A-B│ stands for the absolute value of A-B, so │.5-.6│ would equal .1 (not -.1), and .5 ↔ .6 = 1-│.5-.6│ = 1-.1 = .9.

Examples:	$.5 \leftrightarrow .4$	$= (.5\rightarrow.4) \bullet (.4\rightarrow.5)$
		$[1\text{-}(.5\text{-}.4) \bullet 1$
		$.9 \quad \bullet 1 \;= .9$
	$.5 \leftrightarrow .6$	$= (.5\rightarrow.6)\bullet(.6\rightarrow.5)$
		$= \; 1 \quad \bullet [1\text{-}(.6\text{-}.5)]$
		$= \; 1 \quad \bullet \; .9 = .9$
	$.2 \leftrightarrow .1$	$= (.2\rightarrow.1) \; \bullet (.1\rightarrow.2)$
		$= [1\text{-}(.2\text{-}.1) \bullet 1$
		$= \; .9 \quad \bullet 1 \;= .9$
	$.9 \leftrightarrow .1$	$= (.9\rightarrow.1) \bullet (.1\rightarrow.9)$
		$= [1\text{-}(.9\text{-}.1)\bullet 1$
		$= \; .2 \quad \bullet 1 \;= .2$

In this way fuzzy conditionals are said to be capable of more discriminations, of quantifying shades of gray. The first example for the conditional definition above ($.5 \rightarrow .4$) loses truth in moving from the antecedent to the consequent, but it loses such a little bit of truth that it is closer to being wholly true than completely false, so it receives a .9 degree of truth. A classical interpretation would force us to classify this conditional statement as all or nothing, as false because it is not wholly true. Note that the classical values are recovered when the antecedents and consequents are wholly true or false, showing again that classical logic captures only the extreme end points on a continuum of discriminations. Since a biconditional can be defined in terms of the fuzzy conjunction of two fuzzy conditionals, when the components of a fuzzy biconditional are close in value to each other ($.5 \leftrightarrow .4$, $.5 \leftrightarrow .6$, $.2 \leftrightarrow .1$), a high degree of truth will result (.9); when they are far apart ($.9 \leftrightarrow .1$), a low degree of truth will result (.2).

Although we will not pursue the generalization of classical logic further in this book, quantification logic is also extended in fuzzy logic such that more precise interpretations are given for universal and existential quantifiers. In addition, approximate reasoning terminology found in natural language is also defined so that more quantifiers are used, such as *many*, *few*, *almost*, and *usually*. In this way, fuzzy logic is said to be able to handle fuzzy syllogisms, such as:

> Very old fossils are usually rare.
> Rare fossils are hard to find.
> Therefore, very old fossils are usually hard to find.

A fuzzy interpretation of conditional statements in conjunction with fuzzy set theory is very important for creating and running more energy efficient technology. Typical rules in a fuzzy controlled air conditioning system are: If the temperature is just right, then the motor speed is medium; if the temperature is warm, then the motor speed is fast. But the terms *just right* and *warm* are fuzzy sets; they do not have rigid cut off lines as in Venn diagrams. They each cover a range of temperatures and their ranges overlap and blend together. For instance, a temperature of 73 degrees may be interpreted as .9 degrees within the fuzzy set of just right, but .2 degrees in the set warm. In a fuzzy controlled system, these rules are "fired" together and an average is taken to arrive at a smooth motor speed.[6] The rules are said to fire in parallel and partially. The antecedent of the *if . . . then* rule describes to what degree the rule applies. *Warm* is not an all or nothing quantity. If it was interpreted in terms of just a 0 or a 1, as either being exactly warm or exactly not warm, then the motor speed would stay fast or not fast at an entire range of temperatures.

In Japan, by the early 1990s the famous Sendai subway was already using 59 rules. They all fired constantly to some degree. The subway quickly became known as one of the smoothest riding and energy efficient in the world. Soon fuzzy controlled pocket cameras were being produced using approximately 10 fuzzy rules to control autofocus. Hitachi, Matsushita (Panasonic) in Japan, and Samsung in Korea developed fuzzy washing machines using approximately 30 fuzzy rules to relate and smoothly control load size, water clarity, and water flow. Sanyo Fisher's 8mm fuzzy video FVC-880 camcorder used nine fuzzy conditionals. Professor Michio Sugeno at the Tokyo Institute of Technology developed a fuzzy control system for a helicopter that was capable of feats that no human could match, nor any previous mathematical model. Hitachi, Matsushita, Mitsubishi, Sharp, and Samsung developed fuzzy air-conditioner controllers that purportedly saved 40% to 100% in energy. Sony developed a palmtop computer that recognized handwritten Kanji characters, and Sharp developed a prototype of a refrigerator that would be capable of learning a user's pattern of usage and adjusting defrosting times and cooling times accordingly. By the early 90s, Goldstar, Hitachi, Samsung, and Sony were all working on perfecting TV sets that would be able to adjust volume depending on the viewer's room location. Maruman even developed a golf diagnostic system.[7] In 2001 GE introduced a fuzzy controlled front loading washing machine that “Cares for Wash INTELLI – GENTLY." It was advertised to sense the weight of the clothes and water temperature to determine the best wash time, with the least energy used, along with a special tumbling action that would clean clothes gentry and thoroughly with the least amount of wear and tear on the clothes. By that time, the Lord of the Rings trilogy used fuzzy logic controllers in the MASSIVE 3D animation software.

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[6] The motor speeds of medium and fast are also interpreted as fuzzy sets.

[7] For a readable summary of the state of fuzzy product development by 1993, see Bart Kosko, ***Fuzzy Thinking: The New Science of Fuzzy Logic*** (New York: Hyperion, 1993), pp. 180-190. Kosko also notes that by the early 90s Japanese firms held over a thousand fuzzy patents world wide and 30 of the 38 in the U.S. For a more recent summary of the state of the art, see ***Fuzzy Logic Applications in Engineering Science (Intelligent Systems, Control and Automation: Science and Engineering)***, J. Harris (Springer, 2005).

Resolution of Paradoxes and Implications

As our air conditioning systems are more efficient using multivalued logical programming, so can we better understand the sorites paradoxes. From a classical point of view, the above paradox involving height seemed technically sound. The classical valid rule of modus ponens was correctly applied, and there appeared to be no case where we could declare that one of the premises was false—no case where an antecedent was crisply true and the consequent crisply false. If person 15 is short, then person 16 has to be short, since person 16 is only 1/32 of an inch taller, and so on.

In fuzzy logic, the resolution of the paradox comes by first realizing that we no longer need to commit ourselves to the crisp truth of all the premises. Because a degree of shortness will be assigned to each person, a lesser degree of shortness and hence truth will be assigned to each person as we move up the sequence. If the person 5 feet tall is assigned a 1, then the person 5 feet 1/32 of an inch will be assigned .99. Thus, the very first *if . . . then* premise under a fuzzy conditional interpretation is not wholly true: $1 \rightarrow .99 = 1\text{-}(1\text{-}.99) = .99$. Because there is a loss of truth in moving from the antecedent to the consequent, the conditional is not wholly true. Second, we note that given our generalized notion of validity, modus ponens is no longer always wholly valid! If we demand that a 100% valid inference should be inconsistent with any loss of truth in moving from premises to conclusion, then there will be instances were modus ponens fails this requirement. Consider the following expected results where fuzzy sets are mixed with a classical interpretation of modus ponens:

1.	$1 \rightarrow .99$ $1 \quad / \therefore\ .99$	**2.**	$.99 \rightarrow .98$ $.99 \quad / \therefore\ .98$

The first modus ponens is wholly fuzzy valid, but the second is fuzzy invalid. In 1 there is no loss of truth in moving from the premises to the conclusion, but in 2 there would be. That is, in fuzzy logic it is incorrect to say that the value of the conclusion .98 follows from the premises of 2. Here is how this is determined in fuzzy logic:

1	$1 \rightarrow .99 = 1\text{-}(1\text{-}.99) = .99$ $1 \qquad = 1 \ / \therefore\ .99$	(The conclusion = minimum value of the premises.)
2	$.99 \rightarrow .98 = 1\text{-}(.99\text{-}.98) = .99$ $.99 \qquad = .99 \ / \therefore\ .99$	(The conclusion = minimum value of the premises.)

Since in fuzzy logic the value of the conclusion equals the minimum value in the premises, the correct assignment of value in 2 should be .99 for the conclusion, not .98. The value of .98 returned by modus ponens involves a loss of truth, so the classical interpretation of 2 is not consistent with a generalized notion of validity using fuzzy sets that demands that there be no loss of truth in moving from the premises to the conclusion. At best the classical version of modus ponens is now seen as a "weakly valid sequent"; it remains wholly valid only at the end points of a sequence of discriminations, where the antecedent and consequent have the values 1 or 0 or values very close to 1 or 0 as in case 1. **In a very important sense, in fuzzy logic a valid argument can blend into an invalid argument**. Between the end points of 100% valid and 100% invalid, there can be degrees of validity and invalidity in between. Modus ponens is wholly valid only some of the time.[8]

Bottom line: **Just as there is a range of degrees of truth for statements between the extremes of 100% true and 100% false, so there are degrees of validity between 100% valid and 100% invalid**. In our height argument paradox above, as we move through the inferences from Midge to Kareem, the inference becomes less and less valid. Consider:

If Midge (five feet tall) is short, then Bea (5 feet 1/32 on an inch) is short.
Midge is short. So, Bea is short.

$1 \rightarrow .99 = 1\text{-}(1\text{-}.99) = 1\text{-}.01 = .99$
$1 \qquad\quad = 1 \ /\therefore\ .99$

If Midge (five feet tall) is short, then Jorge (seven feet five inches) is short.
Midge is short. So, Jorge is short.

$1 \rightarrow .03 = 1\text{-}(1\text{-}.03) = 1\text{-}.97 = .03$
$1 \qquad\quad = 1 \ /\therefore\ .03$

Now the paradox disappears and our reasoning returns results consistent with our common sense. An inference that if Midge is short to Bea is short is reasonable. But an inference that if Midge is short, then Jorge is short is not.

We can also show that $(S \supset {\sim}S)\bullet({\sim}S \supset S)\ /\therefore\ S \bullet {\sim}S$ is no longer 100% valid in fuzzy logic. Consider the case where S = .4. The following results with a fuzzy interpretation:

$(S \rightarrow {\sim}S) \bullet ({\sim}S \rightarrow S)$	$/\therefore\ S \bullet {\sim}S$
$(.4 \rightarrow .6) \bullet (.6 \rightarrow .4)$	$/\therefore\ .4 \bullet .6$
$1 \bullet [\,1\text{-}(.6\text{-}.4)]$	$/\therefore\ .4$
$1 \bullet .8$	$/\therefore\ .4$
$.8$	$/\therefore\ .4$

There is a loss of truth in moving from the premise to the conclusion. There is no loss of truth if

[8] However, in fuzzy logic literature there are efforts to supply a ***generalized notion of modus ponens*** where arguments such as the following would be valid: "Visibility is slightly low today. If visibility is low then flying conditions are poor. Therefore, flying conditions are slightly poor today."

and only if the crisp values of 1 or 0 are given to S. Furthermore, the derivation of S • ~S fails in fuzzy logic, because the rule of Implication is not fuzzy valid -- $(\sim S \rightarrow S) \neq (S \vee S)$. Thus, S cannot be derived from $\sim S \rightarrow S$. If S is .5, we have

$$\begin{array}{lll} (\sim S \rightarrow S) & /\therefore & S \vee S \\ .5 \rightarrow .5) & /\therefore & .5 \\ \quad 1 & /\therefore & .5 \end{array}$$

Only when S is a 1 or a 0, will there be no loss of truth from the premise to the conclusion.

Nor are contradictions a disaster for fuzzy logic. In fact, they are considered normal. Anything that can be placed in a fuzzy set also has membership in other sets partially. Remember that if the temperature is .8 just right, then it could also be .2 not just right or warm. Like a Buddhist, the fuzzy logician does not see the world as a collection of crisply separated objects, but rather as an ocean of blended drops of water, where occasionally individual drops of water may spray loose from the ocean causing an illusion of separateness. Furthermore, in fuzzy logic everything does not follow from S • ~S, because S • ~S is only wholly false when S is a 1 or 0. In fuzzy logic, fuzzy contradictions cannot have a value higher than .5, but they can have any value between 0 and .5 However, if S is .5, then S • ~S equals .5, and any value less than .5 will not follow as a fuzzy valid sequent. If A is .4 and S is .5, then "S • ~S /∴ A" will have a loss of truth.

The most important assumption in Aristotelian logic is Av~A. Everything must either be or not be. This principle is called the ***Law of Excluded Middle*** and is seen as the ultimate foundation for Western culture's pursuit of truth. We may disagree on many things, but don't we at least know for sure that something either exists or it does not exist? We may not know yet whether our solar system has another, presently unseen, planet, but surely we know ahead of time that another planet either exists in reality or it does not. Hence, the little killer proof B /∴ Av~A in chapter 10 was very important. It shows that Av~A is always true, so it can be derived from any statement.

From a fuzzy logical point of view, this allegiance to Av~A is viewed as something like a cultural Questionable Dilemma fallacy. As we have seen, in fuzzy logic not all contradictions are the same; some can have a degree of truth as high as .5. Similarly, the so-called Law of Excluded Middle can have a degree of truth as low as .5. In fuzzy logic De Morgan's theorem, Double Negation, and Commutation still hold, so $\sim(A\bullet\sim A)\leftrightarrow(A\vee\sim A)$ regardless of the degree of truth of A. But if A equals .5, then $(A\bullet\sim A)=(A\vee\sim A)$. This is the ultimate slap in the face for Western logic: what were formerly thought to be contradictions and tautologies dissolve into each other. In the late 1980s and early 1990s considerable controversy focused on fuzzy interpretations of contradictions. Editors of major professional journals refused to publish articles that asserted that A•~A could be something other than totally false.

Philosophy: What about reality?

In the 1990s one of the first leading proponents of fuzzy logic outside of Asia was Bart Kosko. With degrees in philosophy, mathematics, and electrical engineering, he often made fun of people who thought that philosophy is a worthless degree. In his book, ***Fuzzy Thinking: The New Science of Fuzzy Logic***, he recounts how he often turned to philosophy for guidance in discovering new mathematical and logical relationships to solve engineering problems. Repeatedly in this book we have noted the connection between philosophy and the foundations of logic and the very practical matters of technological development, belief acceptance, ethics, and meaning in life. In this chapter we have seen how philosophy literally can be "cashed" into technology and billions of dollars in product development.

In other words, the apparent ivory tower debates of philosophers can be of great consequence. In spite of how much the world has changed in terms of multicultural connections, many Western philosophers, for the most part, still have little respect for Eastern philosophers, and vice versa. Many Western philosophers see themselves as rigorous, disciplined, and scientific. Eastern philosophers are thought to be wishy-washy, vague, and incoherent. Eastern philosophers see Western philosophers as dogmatic, ideologically blind, and culturally egocentric. People who try to do what is called Comparative East-West philosophy are too often scolded by both Western and Eastern philosophers for their lack of insight or weak standards of inquiry.

In this context the whole notion of fuzzy logic has created passionate debate. Consider a few famous quotes:

> Fuzzy theory is wrong, wrong, and pernicious. What we need is more logical thinking, not less. The danger of fuzzy logic is that it will encourage the sort of imprecise thinking that has brought us so much trouble. Fuzzy logic is the cocaine of science. Professor William Kahan, University of California at Berkeley[9]

> 'Fuzzification' is a kind of scientific permissiveness. It tends to result in socially appealing slogans unaccompanied by the discipline of hard scientific work and patient observation. Professor Rudolf Kalman, University of Florida at Gainesville[10]

The influential Harvard philosopher and logician Willard Van Orman Quine once described alternatives to classical bivalent logic "deviant," and branded some of the literature on fuzzy logic "irresponsible."[11] Furthermore, critics of fuzzy logic will claim that it is simply regular

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[9] Quoted from Lotfi Zadeh, "Making Computers Think Like People," ***IEEE Spectrum***, August, 1984, p. 4. Kahan made these statements in 1975.

[10] Quoted from Daniel McNeill and Paul Freiberger, ***Fuzzy Logic*** (New York: Simon & Schuster, 1993), pp. 46-47.

[11] W. V. Quine, ***Philosophy of Logic***, 2nd ed., (Cambridge, Mass.: Harvard Univ. Press, 1986), Chapter 6 and page 85.

logic in disguise and that new intelligent product development is due more to improvements in sensor technology than the Buddha. There are no alternate rationalities: modus ponens is as valid in China as it is in the United States. For instance, basic rules of inference were used in developing the above presentation of fuzzy logical connectives basic rules of inference were used. Take one example of the Extension Principle. In defining fuzzy conjunction did we not reason as follows?

> If a classical conjunction returns the minimum truth value of its parts (T•F = F), then a fuzzy conjunction will return the minimum truth value of its parts (.8•.2 = .2). A classical conjunction returns the minimum truth value of its parts (T•F = F). So, a fuzzy conjunction will return the minimum truth value of its parts (.8•.2 = .2). Modus Ponens

Is fuzzy logic simply a gimmick? A shallow amendment to classical set theory? A surface dazzle that, when uncovered a little, shows regular logic at its core?

Fuzzy proponents will respond by reminding their critics that fuzzy logic expands rigor. To repeat the metaphor used throughout this book, the claim is that we are still following reasoning trails but the paths are now broader. Fuzzy proponents will also claim that they are being more scientific than the critics of fuzzy logic. Classical logic as a foundation for producing expert systems and computers with artificial intelligence (AI), computers that can think intelligently and learn, has been tested and failed. A true scientist must be empirically honest. If your pet theory fails lots of tests, then you must give it up regardless of the effort invested. According to Kosko,

> After over 30 years of research and billions of dollars in funding, AI has so far not produced smart machines or smart products. . . . The AI crowd . . . took state funds and defense-buildup funds and set up their own classes and conferences and power networks. And they did not produce a single commercial product that you can point to or use in your home or car or office. They beat up on fuzzy logic harder than any other group, because they had the most to lose form it and the fastest. Fuzzy logic broke the AI monopoly on machine intelligence. Then fuzzy logic went on to work in the real world.[12]

According to Lotfi Zadeh, the University of California at Berkeley professor who first proposed the principles of fuzzy logic in the 1960s, trying to produce AI with bivalent logic is "like trying to dance the jig in a suit of armor."[13] Classical logic as a foundation for AI has failed and it is only dogma and inertia that is keeping it around. According to Kosko, in the early days of fuzzy logic, "It was daring and novel . . . because you first had to get your university degrees in the old black and white school and then doubt that school and rediscover what any layman could have

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[12] Kosko, 1993, p. 159-60. Note that in addition to an inductively correct falsification point, Kosko commits an ad hominem circumstantial here.

[13] Quoted from Kosko, p. 160. A questionable analogy or simply an introduction to an argument?

told you about common sense—it's vague and fuzzy and hard to pin down in words or numbers."[14] Any average human being knows what words like *many, few, almost, a little, a lot, usually,* and *quite true, very true, more or less true, and mostly false* mean. Computers can not be smart unless they can compute these notions.

Quantum Logic and Quantum Computers

To conclude this book let's end with one of the most intriguing and controversial issues that developed from the physics of the twentieth-century—the nature of reality. Even a defender of fuzzy logic could adopt a purely pragmatic stance and argue that although fuzzy logic works, we need not talk about multiple realities or endorse a Buddhist ontology[15] that says that separate objects are illusions and that reality is one. To define a fuzzy set *short* in terms of degrees of shortness is a far cry from saying that my short friend John is one with the universe, that his apparent individuality is an illusion, and that the apparent physical world of separate objects is an illusion as well. A pragmatic defender of fuzzy logic could still argue that Aristotle was right. John is either here now or he is not here. He can't be here (in Hawaii) and also be in New York at the same time.

However, one of the stimulants of fuzzy logic was the twentieth-century development of what is called quantum physics. You have no doubt heard many times the word *quantum*, as in quantum jump and quantum leap. (I have even seen beauty ads for quantum perms.) In spite of the term's wide use, few people understand just how radical the notion of a quantum jump is. Quantum physics has been very successful. It is the basic physics of how the atom and its parts work, and many of our present chemical and electronic technologies are based on it in one way or another. Ever wonder how so much information (texts, web pages, and video) can show up in your smart phone? We use quantum physics every day, but its math is weird. The math describes electrons for instance as jumping around all the time. But they don't jump around like you and I can jump around. When they jump from point A to point B, they are nowhere in between. They are not distinct objects that move in a continuous space; it is not just that they jump fast, so fast that we can't detect them; they don't exist in between the points. The math, if taken literally (a big "if"), says that they pop in and out of existence. Worse, if the math is taken literally, a degree of each electron around every atom in the universe is a little bit everywhere. So, some of John is in Hawaii and New York at the same time.

Quantum physicists also talk about states that show the phenomena of *entanglement* and *supposition*. Photons of light can be sent in different directions with different spin orientations

[14] Ibid., p. 161.

[15] ***Ontology*** is a technical philosophical term that means in this context "theory of reality," a theory about what exists, about what is most fundamentally real.

(polarization), but famous experiments have shown that it is a mistake to picture these photons as independent little things with a distinct individuality with distinct spin properties. Until observed the state of entangled “photon-ness” is in a supposition of all possible spin orientations. Imagine a cat that is both dead and alive or a person who is simultaneously both short and tall.

In their college educations many older physicists and engineers have been taught to ignore what the math says literally. Their educations have been heavily influenced by ***logical positivism***, a philosophy popular in the early to middle decades of the twentieth century, and a philosophy of which proponents of fuzzy logic are very critical. Logical positivism said, "Never take the math seriously in terms of what it says about reality; the job of a scientist is to adequately link experiences, not to tell us what is happening behind the scenes of those experiences." Logic and experience are all that a rational person should worry about. Questions regarding the nature of reality are on par with questions about whether God exists and the meaning of existence. Objective closure on such questions is not possible. No hard evidence can ever be achieved regarding answers to such questions. In other words, the math is just a practical human tool that we use to predict results in scientific experiments and ultimately to develop technology that works.

For a simple example, recall that in the hat example in Chapter 1, we could predict that there were two hats in the closet (5-3=2). However, it is a mistake to equate mathematics with reality, say the logical positivists. Mathematics is a vast set of practical tools and different tools have to be used for different situations. 2 + 2 is not always four. Placed at the same place on a pane of glass, 2 drops of water and 2 more drops of water will not make four drops of water!

The way this works out in terms of subatomic particles, such as quantum jumping electrons, is that the mathematical function that describes the electron as being "smeared out" throughout the universe is viewed as a statement of probability, as a prediction in degrees of probability of where we will find an electron when we look for it. Interpreted this way, an electron moving rapidly around an atom within an object in Hawaii has a certain probability of being in New York or the Andromeda galaxy, but these probabilities are very, very low compared to the probability that we will find the electron in Hawaii very, very close to the nucleus of its atom.

Otherwise one can view the electron as literally being everywhere and then undergoing a quantum "collapse" immediately to a specific point in Hawaii, and observing one leg in the experiments on polarized light beams is viewed as collapsing the supposition of states into distinct photons with distinct spin orientations, even if the separated photons are millions of light years away! Even more extreme, if one takes the math very seriously, when we observe the electron in one place, we and the electron are actually in all the other places as well but are in different universes! The latter interpretation is known today as the Many-Worlds interpretation of quantum mechanics. It was proposed as a PhD thesis by Hugh Everett III at Princeton University in the 1950s. Everett’s original proposal was called *Wave Mechanics Without Probability*, but to pass his degree requirement Everett was urged to tone down the radical ontological implications of his mathematical work.

Logical positivism said that these realistic interpretations were absurd, that it is absurd to believe that we have evidence that reality behaves this way. Furthermore, we need not even contemplate what the real electron is doing. They told scientists just to get on with the task of building twentieth century technology.

In this way the entire edifice of bivalent logic was saved from potential demolition by quantum physics. We could continue to go about our bivalent Western ways and ignore that the behavior of electrons (and all subatomic reality for that matter) seemed to say that Aristotle was wrong, that electrons could be both here and not here. Although there were a few outcasts, such as Everett and the physicist David Bohm, who argued that quantum physics was telling us something fundamental about reality, and that that something was closer to Eastern philosophical beliefs than Western, for the most part twentieth century physicists and engineers obediently followed their bivalent teachers who in turn were following the logical positivists.[16]

In general, supporters of fuzzy logic are very critical of probability interpretations. For instance, suppose a doctor tells her patient that there is a good chance a tumor is cancerous because it is quite large. The phrase *good chance* expresses probability and a degree of uncertainty about what is real. However, *quite large* describes reality and is not an expression of uncertainty, even though the term is not precise. Kosko does not mince words, "The ultimate fraud is the scientific atheist who believes in probability. . . . the Buddha wins . . . . The universe is deterministic but gray."[17] Supporters of fuzzy logic say that probability is a cop-out, an example of ivory tower faith-healing, a tactic of philosophical ostriches sticking their heads in the sand afraid to face gray reality. We can't ignore that gray reality happens all the time. Probability is the timid measuring of the likelihood of something happening; fuzzy procedures measure ***the degree to which it is happening***. Quantum physics is gray reality big time. The whole universe is in every part; to a degree each object is in every other object.

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[16] David Bohm was a U.S. physicist who worked with Robert Oppenheimer on the atomic bomb. When Oppenheimer was attacked during the 1950s as a possible communist sympathizer, Bohm was so disgusted with his country's behavior he left the U.S. to live in Great Britain. For his philosophy and interpretation of quantum physics, see his book, ***Wholeness and the Implicate Order*** (London: Routledge & Kegan Paul, 1980). For some interest in Bohm's interpretation, see "Bohm's Alternative to Quantum Mechanics," by David Z. Albert, ***Scientific American***, May, 1994, pp. 58-67. It is interesting to note that this same issue of ***Scientific American*** had an advertisement (p. 3) for a Mitsubishi Galant, hawking the new "intelligent shifting of a Fuzzy Logic transmission." This same ad could be found in other popular magazines by 1994. In the U.S. ads in the early to mid 90s like this represented somewhat of a marketing breakthrough. Advertising executives had been worried that the general public would view the adjective 'fuzzy' negatively. Here is an example of a transitional ad, in this case for a Saturn SW2: "A 124-horsepower dual-overhead-cam engine linked to an automatic transmission utilizing fuzzy logic programming. (Huh?) It gives a Saturn the ability to adjust to different driving conditions—optimizing performance and handling. Still fuzzy on it? Any Saturn sales consultant would be more than happy to clarify things." ***Scientific American***, October, 1994, p. 51.

[17] Kosko, 1993, pp. 50, 63.

Many Western philosophers and scientists for the most part say this is absurd. It's mysticism and opens the entire structure of scientific rationality to occult and paranormal silliness.[18] If Western logicians are bit-brains, fuzzy logicians are flip-brains (from ***fuzzy logical inferences per second***). Fuzzy supporters counter that an extension of fuzzy principles will open up entire new ways of seeing difficult ethical questions, human nature, God, and meaning in life.

We cannot follow these debates further. According to Rudyard Kipling, "Oh, East is East, and West is West, and never the twain shall meet." If Kipling is correct, Aristotle and the Buddha will never get along. On the other hand, if the supporters of fuzzy logic are right, they are already getting along inside the smartest of our new computers. New computers are using chips that do both good old fashioned bit processing and flip processing. The latter are combined with what are called neural nets, a process of computing that is said to mimic the way the human brain works. One thing seems certain: The twenty-first century promises to be very interesting philosophically and technologically.[19]

What we can do is give a brief summary of the logic and technology that is being worked on in spite of the different philosophical interpretations of quantum physics.

Quantum logic makes a major revision in some of our most basic propositional rules. For instance, the rule of Distribution is no longer a tautology. Because of entanglement supposition states and the famous quantum uncertainty principle—we cannot know the precise location of a quantum object (p) and its precise mass and speed (momentum, q) at the same time—p • (q v r) can be true, but (p • q) v (p • r) is false in quantum physics.

To make a long story short, the logical implications of quantum physics have profound implications on the possibility of a new computer revolution. Recall that a major theme of this book has been the recognition that the logical decisions we make at the most basic level have profound implications up the logical line so to speak. Once we commit to a particular definition of *not*, *and*, and *or*, the logic of truth tables, propositional and quantification logic unfold. The most basic assumption in classical computing is very Aristotelian. The fundamental building block of all calculations is the bit, short for binary digit. Think of a bit as being the smallest unit of information. Most important is that a bit can be in only one of two states, 0 or 1, analogous to something being true or false, or a yes or no answer to a question. By processing the answers to

[18] During this time when the opponents of fuzzy logic really got upset, in private conversations they would fire away with ad hominem circumstantial attacks, pointing out that Kosko was from California and that fuzzy logic was first developed at Berkeley. Thus, implying that fuzzy logic should not be considered seriously because it originated in a state that was a hotbed of new wave fads that a more disciplined person laughed at.

[19] For another introduction to fuzzy logic for the nonexpert, see ***Fuzzy Logic***, by Daniel McNeill and Paul Freiberger (New York: Simon & Schuster, 1993). For technical articles, see ***Fuzzy Logic for the Management of Uncertainty***, edited by Lotfi A. Zadeh and Janusz Kacprzyk (New York: John Wiley & Sons, Inc., 1992), and ***IEEE International Conference on Fuzzy Systems***, March 8-12, 1992.

lots of questions very rapidly—yes (1), no (0), yes (1), yes (1), yes (1), no (0), no (0), and so on—modern computers can process an enormous amount of information and even appear to make decisions and have insights that exceed human levels of intelligence. However, even in the cases of supercomputers that now routinely beat grandmasters at chess and the best players on Jeopardy, although there is enormous creativity involved in computational design in processing the enormous flow of bits, at the most fundamental level all the computers are doing under the hood is answering yes or no billions or now even trillions of times per second. Watson, the IBM computer that beat the best Jeopardy players in the world in 2011, could sort through 200 million pages of information and answer a question in three seconds. As for the computational creativity part, according to IBM,

> Watson is an application of advanced Natural Language Processing, Information Retrieval, Knowledge Representation and Reasoning, and Machine Learning technologies to the field of open-domain question answering. At its core, Watson is built on IBM's DeepQA technology for hypothesis generation, massive evidence gathering, analysis, and scoring.[20]

As Steve Jobs famously noted in *Playboy* magazine in 1985, "[Computers take] these very simple-minded instructions—'Go fetch a number, add it to this number, put the result there, perceive if it's greater than this other number'—but executes them at a rate of, let's say, 1,000,000 per second. At 1,000,000 per second, the results appear to be magic."

Today think of many trillions per second. Computer speeds are now measured in flops (floating point operations per second) and by 2011 supercomputers routinely achieved speeds of many teraflops and some even the petaflop range. In late 2011, a Chinese made computer was able to perform 1,000 trillion calculations per second. A quadrillion is a 1,000 trillion. So a petaflop is a quadrillion floating point operations per second. Watson had a combined capability of only 80 teraflops. By late 2012 the United States had caught up and surpassed the Chinese computer with the $100 million Titan Cray XK47 at Oak Ridge National Laboratory. It achieved 17.59 petaflops per second. China responded by June 2013 with a computer called Tianhe-2, capable of 33.86 petaflops per second or 33,860 trillion calculations per second. By the time you read this paragraph, some group some where will surely have surpassed this computational ability.

However, even at these great speeds it is estimated that a modern supercomputer would take trillions of years to factor a 1000 digit number. Think about it. For example, how long would it take you to figure out the factors for 42,189? How long would it take to come across 123 x 343? Although there are mathematical processes (algorithms) for finding factors of numbers, for very large numbers much of the process, although systematic, still involves brute trial and error. Computers of course can run through experimental steps and try various combinations of numbers very quickly. It still took several mathematicians using hundreds of computers two

[20] IBM's description of its DeepQA project currently at: http://www.research.ibm.com/deepqa/faq.shtml#3

years to factor a 232-digit number in 2009.[21]

Enter quantum reality and the *qubit*, short for quantum bit. Recall that a quantum object (electron, photon, etc.) can be in a supposition of states. Whereas the Aristotelian bit must be in the crisp 0 or 1 state, a qubit can be in a supposition of both states. Following Everett, one can even think of a parallel universe where in one universe the state is 0 and the other it is 1. Recall in creating truth tables how the possibilities increase exponentially given the number of variables. With two variables, p, q, we have four possibilities of T or F, 0 or 1; with three variables, eight possible combinations of T or F, 0 or 1; with four variables, sixteen . . . with say twenty-five we would have over thirty-three million possibilities, and so on. Imagine then that rather than envisioning each T or F row as a possibility, we have them all happening at the same time. Putting together many qubit calculations as we do now with bit calculations, trillions of possibilities can be examined in trillions of parallel computational universes. Theoretically this implies a calculation parallelism capability far surpassing classical computational speed. The trillions of years for a 1000 digit number would take as little as 20 minutes on a future quantum computer. Regardless of whether one really thinks simultaneous calculations are being made in parallel universes, many contemporary computer scientists are seriously committed to the possibility of a quantum computer and making progress on one that can perform a number of calculations simultaneously.

Although to date only small scale demonstration proof of concept systems have been built—and a maximum of only 10 qubits—optimists believe that we are only a few decades away from a scaled-up system that can contain and make use of large numbers of qubits. Physical processes exist for putting electrons and photons into supposition and entangled states—quantum dots, lasers, nuclear magnetic resonance, shielding nuclear processes in temperature close to absolute zero, and even some day perhaps the manipulation of coffee molecules. The technological trick will be keeping our macroscopic thumbs so to speak from collapsing the supposition states too soon. Quantum suppositions tend to last only a small fraction of a second before a "decoherence" results from interactions with macroscopic objects and processes. Think of decoherence as a quantum bug. Whether this hurdle is only technological or there is a fundamental theoretical barrier is a hotly debated question today.

Again the ontological question beckons. Does the math point to real possibilities, not just mental possibilities, nice logical trails through imaginary space, but realities we can interact with in some way and use? It seems that a younger generation of physicists, mathematicians, and computer scientists are far removed from being influenced much by the ontological timidity of logical positivism. They routinely develop demonstration systems for new ways of creating qubits, publish articles in advanced science journals,[22] and even create quantum information

[21] If you are wondering why anyone would care about the factors of large numbers, think about the security protection you hopefully have the next time you bank online or use a credit card. Sending encrypted data depends on the enormous difficulty of factoring large numbers.

[22] Munro, W. J., Nemoto, K., and Spiller, T. P., "Weak nonlinearities: a new route to optical quantum

centers.[23]

To end this book, it is worth noting that many physicists take seriously the possible reality of multiple universes. In 2011 the prominent theoretical physicist and popular science writer Brian Greene summarized the state of modern theoretical physics in ***The Hidden Reality: Parallel Universes and the Deep Laws of the Cosmos***. A major theme of his delightful book is that a thorough examination of the mathematical trails that stem from all of the present attempted solutions to the puzzles in physics and cosmology all seem to point to one astounding conclusion. Our universe is part in one sense or another of a multiverse of parallel universes.

We know that the expansion of our universe that began about 14 billion years ago is now accelerating at a faster pace than expected for the age of the universe, and that the enhanced acceleration started about 8 billion years ago as the apparent mysterious inflationary force called dark energy overcame the contracting and braking force of gravity. Why? We know—and make magical use of this knowledge everyday with our smart phones and WiFi connected tablets and portable computers—that electrons and photons have particular properties. But why do they have these properties? Why does an electron have a negative charge and a proton a positive charge? Why is an electron more than a thousand times smaller than a proton?[24]

As Greene summarizes, since about the 1960s physicists have been trying to fulfill Einstein's dream of a grand unified physical theory of everything that would not only explain all the unexplained, just-is parameters of our universe, but provide an elegant assimilation of the very large and very small (the cosmic realm of gravity and the subatomic realm of quantum objects) under one mathematical roof. He shows that in following the mathematics of the various solutions we can "trace a narrative arc through nine variations on the multiverse theme." According to Greene,

> . . . the pattern is clear. When we hand over the steering wheel to the mathematical underpinnings of the major proposed physical laws, we've driven time and again to some version of parallel worlds.

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computation," ***New Journal of Physics***, May, 2005. Mariantoni, M., Martinis, J., et al., "Implementing the Quantum von Neumann Architecture with Superconducting Circuits, ***Science***, published online September 1, 2011. Bacon, D. and van Dam, W., "Recent Progress in Quantum Algorithms," ***Communications of the Association of Computing Machinery***, Vol. 53 No. 2, Feb. 2010, Pages 84-93.

[23] Berkeley Quantum Information Center, at which you will find schedules that list "Quantum Lunch Seminar (Vazirani group)."

[24] Best to think of masses, and not sizes literally, because in the standard picture (the standard model) of subatomic objects all particles are viewed as point-particles. This causes lots of problems (calculations that result in infinite numbers) when trying to make quantum theory work with a theory of gravity. It is also hard on our common sense. Atoms have a size but what they are made of do not! Atoms are also mostly empty space. Worse, according to the famous physicists Werner Heisenberg who made major contributions to quantum physics, "Atoms are not things."

For instance, one of the most worked on and hopeful new theories is called string theory, where the point-particle picture of subatomic objects is replaced with a geometry of vibrating strings of energy moving in hidden multidimensional spaces. But in examining the geometry of the multidimensional spaces, one discovers that the one (not yet known) that may produce our universe and its particular characteristics—the particle parameters, the strength of the gravitational and inflationary forces, and rate of cosmic expansion—may be caused by just one multidimensional special geometric twisting, turning, and enfolding out of 10^{500} possibilities! Could each possibility be another universe with different parameters? As Greene notes, it would be surprising if you went into a shoe store at random in a large city and found that the store carried only your size. You would surely want an explanation. But it would not be surprising at all to find your size if you went into a normal store that carried a range of normal sizes. So if there are 10^{500}, the mystery ends (at the expense of many new ones) when we realize that our universe is only one shoe size out of many so to speak.

One of the nine possibilities of a multiverse constitutes the ultimate slap in the face to the logical positivist view on mathematics. Perhaps all mathematics is real. The normal view is that there is the realm of mathematical trails in human minds, or that there is an enormous realm of possible mathematical relationships and many not yet discovered by any human being on the one hand, and then there is reality. The job of science has been to figure out which discovered/created mathematical products of the human mind, or discovered relationships and trails from the possibility realm, best match reality to be used as practical tools for interacting with the real world. In other words, assumed is that only a small portion of all the possible mathematical relationships work in resonating with an independent physical reality. We try lots of possibilities but most fail. However, perhaps this is wrong. Everything that can be thought of mathematically and everything possible mathematically can be thought of at all because in some universe of another the math is functional in that universe. Just as the forces of gravity and inflation, as well as the mass of an electron and other subatomic particles, might be different in a parallel universe, so a mathematics that fails in ours may work just fine in another universe.

In his book, Greene cautions his readers who are probably rolling their eyes over such outlandish theories. The history of the progress of science reveals an important insight. The rules for what constitutes science can expand with our knowledge of what is possible. The rules of good science are not static religious commandments. We can discover good objective reasons to expand our rules. We see also that consistent with a major theme of this book, logical and mathematical rules are not absolute static commandments. We learn, grow, and enlarge our experiential windows, and in the process also expand our practical tools and rules for ever more fascinating reasoning trails.

Exercises I

Using the fuzzy interpretations of the logical connectives, figure out the degree of truth for each of the following. Assume that **A = .7**, **B =.3**, and **C = .1**.

1. $A \bullet B$ 2. $A \vee C$ 3. $A \rightarrow B$ 4. $A \leftrightarrow B$

5.* $(A \bullet B) \vee \sim C$ 6. $B \rightarrow \sim(A \rightarrow C)$ 7. $(A \bullet B) \leftrightarrow C$

8. $\sim(A \bullet B) \rightarrow C$ 9. $A \rightarrow (B \bullet \sim B)$ 10. $\sim C \rightarrow \sim B$

11. $[(A \bullet \sim B) \rightarrow \sim C] \leftrightarrow [\sim(A \vee B) \rightarrow C]$

12. $\sim\{[A \rightarrow \sim(B \rightarrow C)] \bullet [\sim C \rightarrow (\sim A \bullet \sim B)]\}$

13. $A \rightarrow [(A \rightarrow B) \rightarrow B]$ 14. $[(A \rightarrow B) \bullet (B \rightarrow C)] \rightarrow (\sim C \rightarrow \sim A)$

15. $\{[(A \rightarrow B) \bullet (B \rightarrow C)] \bullet (A \vee B)\} \rightarrow \sim(\sim B \bullet \sim C)$

16. $\sim(A \vee C) \leftrightarrow (\sim A \bullet \sim C)$ 17. $\sim(A \bullet B) \leftrightarrow (\sim A \vee \sim B)$

18. $(\sim A \rightarrow C) \leftrightarrow (A \vee C)$ 19.* $(A \rightarrow C) \rightarrow [A \rightarrow (A \bullet C)]$

20. $[\sim A \vee (B \bullet C)] \leftrightarrow [(\sim A \vee B) \bullet (\sim A \vee C)]$

Exercises II

In this chapter a truth table showed that modus ponens makes a tautologous conditional by conjoining the premises of modus ponens and making them be the antecedent of a conditional with the conclusion being the consequent. Repeat this procedure for the 8 remaining rules of inference. Construct a classical truth table for

$$[(p \supset q) \bullet \sim q] \supset \sim p$$

$[(p \vee q) \bullet \sim p] \supset q$,* and so on.

Exercises III

Given the values of **p = .5**, **q = .4**, **r = .3**, and **s = .2** demonstrate which of the classical 9 rules of inference return a value of 1 and which return a value of less than 1. In other words, using these values give a fuzzy interpretation for

$$[(p \rightarrow q) \bullet p] \rightarrow q$$

$$[(p \rightarrow q) \bullet \sim q] \rightarrow \sim p$$

$[(p \vee q) \bullet \sim p] \rightarrow q$,* and so on.

What does this mean? Based on this test, which rules of inference appear to be fuzzy valid? Which rules would never show a loss of truth in moving from the premises to the conclusion?

Exercises IV

Find a fuzzy interpretation that shows that the following classical equivalences and inferences do not always return the value 1.

1. $(A \leftrightarrow B) \leftrightarrow [(A \bullet B) \vee (\sim A \vee \sim B)]$

2.* $(A \rightarrow B) \leftrightarrow (\sim A \vee B)$

3. $(A \rightarrow \sim A) \rightarrow \sim A$

4. $[(A \rightarrow B) \bullet (A \rightarrow \sim B)] \rightarrow \sim A$

5. $[(A \bullet B) \rightarrow C] \leftrightarrow [A \rightarrow (B \rightarrow C)]$

Exercises V

1. Other than De M., DN, Com., and the first version of Equivalence, what rules of replacement will return the value 1 given any fuzzy interpretation? Write a short essay summary explaining what degrees of truth you tested.

2.* Review the classical proof for B /∴ A v ~A and explain why it fails in fuzzy logic. (What line or lines in the proof fail?)

3. In fuzzy logic what would follow from these premises? Visibility is very low today. If visibility is low then flying conditions are poor. Therefore, today flying conditions are . . .

4. As exercises, in chapters 7 and 10 Dan Derdorf's statement made at the Minnesota Metrodome was used: "It's noisy here even when it is quiet." (Q ⊃ ~Q) In classical logic this implies that it is never quiet at the Minnesota Metrodome, (Q ⊃ ~Q) ⊃ ~Q. Fuzzy logic, of course, would interpret the situation differently. Test fuzzy degrees of quiet (.1, .2, .3, ...9) and show that the result in degree of truth for (Q → ~Q) → ~Q will always be equal to ("fire") the degree of truth of either Q or ~Q. Then write out at least four of your tests values in English, translating Q → ~Q, using modifiers such as 'slightly,' 'almost,' 'very,' 'not very,' 'moderately,' etc.

Answers to Starred Exercises

I.

5. (A • B) v ~C
(.7 • .3) v ~(.1)
.3 v .9
.9

19. (A → C) → [A → (A • C)]
(.7 →.1) → [.7 → (.7•.1)]
1-(.7-.1)→[1-(.7-.1)]
.4 → .4
1

II.

p	q	[(p v q) • ~p] ⊃ q				
T	T	T	F	F	**T**	T
T	F	T	F	F	**T**	F
F	T	T	T	T	**T**	T
F	F	F	F	T	**T**	F

III.

$[(p \text{ v } q) \bullet \sim p] \to q$
$[(.5 \text{ v } .4) \bullet \sim .5] \to .4$
$[\ \ .5 \quad \bullet .5] \to .4$
$.5 \to .4$
$1-(.5-.4)$
$.9$

IV.

2. $(A \to B) \leftrightarrow (\sim A \text{ v } B)$	Many values will work in showing
$(.4 \to .4) \leftrightarrow (.6 \text{ v } .4)$	that this relationship will not
$1 \leftrightarrow .6$	always return the value of 1.
$(1 \to .6) \bullet (.6 \to 1)$	Here A and B both equal .4.
$[1-(1-.6)] \bullet 1$	
$.6 \bullet 1$	
$.6$	

V.

1. B /∴ A v ~A	
2. B v A	(1) Add
3. A v B	(2) Com.
4. ~A → B	(3) Impl.
5. ~A → (~A•B)	(4) Abs.
6. A v (~A•B)	(5) Impl. **X**
7. (Av~A)•(AvB)	(6) Dist.
8. A v ~A	(7) Simp.

The rule of Implication will not always return the value 1. However, step 4 is fuzzy valid. Step 4 (~A → B) will never have a lower truth value than step 3 (A v B). Step 6 is the problem, because the other half of the Implication rule, (~p → q) → (p v q), will not always return the value 1. So, step 6 can have a lower truth value than step 5.