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Universal Logic Applied to a Defeasible World

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Abstract

Universal logic applied to the real world is defeasible. Defeasible reasoning where inferences depend (informally) on implied conditions can be better understood when made formal and indeed needs to be formalised for implementation in computational systems of practical reasoning such as interconnections in globalised eBusiness. Semi-formal Aristotelian syllogisms are now expressed within modern formal methods as Schütte-Ackermann language such as $\Gamma \longrightarrow \phi$. At face value the $\Gamma, \Delta \longrightarrow \phi'$ gives no inconsistency but what if $\phi' = \neg\phi'$? For we cannot then formally derive with *tertium non datur* the defeasible expression $\Gamma, \Delta \longrightarrow \neg\phi$. A deeper examination of these terms including the meaning of the negation $(\neg\phi)$ is required. The very formal topos with a higher-order slice category can provide the level of abstraction to handle rigorously informal concepts like non-monoticity and alternative reasoning in disparate fields with the same pullback. The security of artificial agents in Java software is an example requiring this full understanding of defeasance.

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1 Background

That the real world continues to exist coherently in both space and time suggests (and there are some who would say requires [28]) a universal logic. In work on pure theory the term universal logic is different things to different people. McGinnis [1] deprecates 'a persistent ambiguity in the use of this term' universal logic distinguishing Béziau's *research program (sic)* on mathematical structures in developing a general theory of logics from the use of the term by Routley and Sylvan or Brady 'to denote a *type of logical system* that is universal in the sense that it is supposed to be applicable in every context including contradictory and impossible contexts'. McGinnis may well be right in pure theory to assert that, 'Clearly these are two very different concepts' but here in the applied context of the real world we have to treat universal logic not as 'a type of logical system' but the type of logical system for which Béziau's *research program* comprises

[1] In his review of [7] in [45] at p.450.

structures as subtypes and views, or models of universal logic. For applied logic, there is not the same opportunity for distinguishing ideal views. The ideal world is not reality and the real world is not ideal. Any perceived distinction in structure must all fit together in one universal logic if the world is to hold together.

The deconstruction of the components gives rise to defeasance. Defeasance arises from real-world conditions and is therefore prominent in practical reasoning. Prime examples are legal reasoning [20, 41, 2, 49, 42, 18, 19] and perception like Pollock's OSCAR system of reason-schemas for intelligent agents [48]. It arises in fields other than law for instance in computer security [30], AI with 'common sense' aspects [10], consciousness [48] and cognitive dissonance [2] [12]. Pattern recognition systems can involve forms of alternative reasoning. This may be straight-forward belief revision, for example recognition of a person, "no it's not: it's somebody else" is defeasible reasoning whether perception in a human observer or by intelligent agent in image retrieval.

Interconnections in globalisation are a breeding ground for defeasance. A current practical example is product logic in eBusiness involving intelligent agents which co-operate and react to change, to other agents and to the environment in order to negotiate a route through networks while attaching intelligence to individual orders and products. They can be applied to packaging to give a flexible transport system with the input of any machine as the output of any other machine. A resource agent acts on the instructions of an order agent to deal with product agents. These can be endowed with a buying capacity using electronic cash. Intelligent agents negotiate their way through an entire supply chain involving raw materials, supplies, logistics, manufacturing and distribution without human intervention according to an economic rationality based on electronic trust. The business model of a static pre-defined usually hierarchical planning system is replaced by an *ad hoc* and heuristic negotiation and collaboration between intelligent software modules in eBusiness [35]. The intelligence makes decisions about anticipation at run-time. Defeasance arises from complexity – either data with a complicated structure of many interacting internal relationships or large quantities of simple data where the bulk nature seems to generate its own complexity. These potential systems will need to have defeasible reasoning built into their logic.

Defeasible reasoning from a cognitive science perspective is often described as non-monoticity. Or because the addition of different conditions may provide a different conclusion, the phrase *belief revision* is sometimes found. Another view more from symbolic logic is the *counterfactuals* of Goodman and Lewis [3]. Versions of anomalous reasoning are very often discussed informally or by using semi-formal vocabulary as in the classic example about the bird 'Tweety' [5, 1, 46]: if x is a bird, x flies, symbolised as:

[2] Cognitive dissonance may arise from anomalous perception as in optical illusions or from a strange environment like travelling along a Moebius strip – a common example in the literature. Discomfort raises an emotional counterpart to ontology which will not be pursued here.

[3] [17, 40]. Lewis's book begins with 'if kangaroos had no tails they would topple over'.

$$\frac{x \text{ is a bird}}{x \text{ flies}} \quad \text{or} \quad \frac{\underline{x \text{ is a bird}}}{x \text{ flies}}$$

where the premise or antecedent is over the bar and the conclusion or consequent is under the single bar. The double bar may be used to indicate that the converse also holds. However in this example if x flies, it does not necessarily follow that x is a bird.

1.1 Defeasible Reasoning in Natural Systems

An example of agent defeasance already occurs on the Internet in the form of worms and viruses on some platforms where such malicious codes can be dynamically loaded in the program at run-time. The employment of Java within the Microsoft operating system is particularly prone to breaches of security arising from defeasible conditions. Owing to its dynamic loading feature Java is not type safe [57] and this has led to suggestions for an improved architecture [16]. However a formal analysis by [30] indicates that the facility in Java to load dynamically is fundamental to the language. A solution to this problem of defeasance is the use of 'patches' which are palliatives rather than cures. The dynamic loading problem is a classical case of defeasance with the strengthening of the antecedent in the form of additional code leading to a new consequence.

The correlation between real-world conditions and conditions of logic needs to be explored in their formal representations for a better understanding particularly in the context of defeasance. For implementation in computational systems of practical reasoning such as in law more rigorous formalisation is required. Therefore the mode of inference, the meaning of a conditional or logic gate and even the sense of negation itself have to be examined. An early paper on defeasible reasoning with respect to law was presented at the 21st IVR [27] and further work continues in this area with a view to publication at the 23rd IVR World Congress Law and Legal Cultures in the 21st Century: Diversity and Unity, August 1-6, 2007 Kraków, Poland.

2 Axioms and Inference in Formal Methods

The movement to formalise reasoning and logic which began in Aristotelian syllogism developed in the twentieth century with an emphasis on axioms and derivability. These were the subject of Hilbert's program [29] using 'finitary' methods which assumed completed infinite totalities and so avoided some objections of the Brouwer school of constructivist mathematics. Out of this program has come both proof theory [4] and model theory as separate sub-disciplines of logic in their

[4] The aim of Hilbert's *Beweistheorie* was as a tool to analyse all possible derivations in formal axiomatic systems (([11] at p.49).

own right. But their domain is normally restricted to applications in pure mathematics and the finitary method does not extend to the ontology of a wider field of application as needed in computing science. For although Hilbert's program has led to further examination into the syntax and semantics of truth by workers like Gödel, Carnap, Tarski [62] and Kripke [36], nevertheless foundational studies in pure mathematics [5] do not really call for an examination of pragmatics. It is this aspect which is wanted in real-world subjects like natural language and intelligent agents where issues of defeasibility lie.

Hilbert's program has stabilised within the language of Schütte-Ackermann [59] consisting of sequent calculi of the form $\Gamma \longrightarrow \phi$. Γ is a context of parameters such as a finite sequence of formulae and ϕ is a single conclusion [6]. The notation for the operation [7] of strengthening the antecedent $\Gamma, \Delta \longrightarrow \phi$ is used for the union of the sequents Γ and Δ on the left-hand side with the same conclusion ϕ. In informal classical logic this is an argument *a fortiori* or *abundans cautela* where the addition of sequent Δ is redundant. Alternatively belief revision may reach a full negative conclusion $\Gamma, \Delta \longrightarrow \neg\phi$ or just result in different explanation $\Gamma, \Delta \longrightarrow \phi'$.

The stark result $\phi \leftrightarrow \neg\phi$ cannot be derived even in intuitionistic predicate calculus [8] with the excluded middle [9] but strengthening the antecedent might result in the different conclusion:

$$\Gamma \longrightarrow \phi \quad \text{and} \quad \Gamma, \Delta \longrightarrow \phi'$$

This is a weaker form short of a direct negation. Therefore in the example 'x is a bird x flies': $\Gamma \longrightarrow \phi$. The additional proposition may give 'if x is a bird and x is dead, x is a dead bird that flies': $\Gamma, \Delta \longrightarrow \phi'$. This is rather different.

However the theorems and inference rule of the sequent calculus of Schütte-Ackermann cannot support this reasoning. It does not really matter whether the sequents are propositions or first-order predicates, classical or intuitionistic, and there does not seem any prospect that the position would improve in sequents of modal logic. The problem seems to arise from the combination of the inference with the negation and the background to this should perhaps be first explored further.

The logic connectives $\wedge, \vee, \neg, \Rightarrow$ have a fixed meaning in classical symbolic logic corresponding to the words 'and, or, not, implies'. In natural language, however, these same words have a much more fluid use which are inherent in

[5] These correspond to the foundations and limitations for formal methods in computer science which are divided from practical computing similar to the division between pure and applied mathematics.

[6] The Schütte-Ackermann language treats quantities like Γ and Δ as a set or a linearly ordered sequence. This is reductionist. It would be better to treat these as categories of objects with specific relationships between them.

[7] Described by von Wright [63] at p.23 and discussed by Schmill [58] at p.244 note 3.

[8] for second-order intuitionistic logic see [47].

[9] However, it may be possible in other formalisms like the logical tetralemma of Buddhist logic where apparently negation has a different sense [61].

legal reasoning. A familiar example is the use of 'and/or' interchangeably. From a narrow linguistic viewpoint this indecisive use is sometimes looked on as a weakness ([60], at p.45; [4]) but there may be more to it and this may really be a strength. The comma in $\Gamma, \Delta \longrightarrow \phi'$ is normally described somewhat loosely as a union, presumably implying a disjoint union. However, the example just given well shows that the introduction of Δ may modify Γ. Γ and Δ are not independent 'x is a bird' is subtly altered by the addition of 'and x is dead' [10].

3 Set Logic

Defeasance is quite fundamental. It goes back to Russell's paradox [55] about the set of all sets that are not members of themselves. Such a set, if it exists, will be a member of itself if and only if it is not a member of itself. According to Russell [11]:

$$\frac{\text{the set of all sets is a member of itself}}{\text{the set of all sets is not a member of itself}}$$

this would be in Schütte-Ackerman: $\phi \longrightarrow \neg\phi$. Stripped down to its basic form of defeasance, we can see that the strengthening of the antecedent may be a red herring. The problem can arise from the negation independent of any additional conditions. Russell's own solution to his paradox [12] was to use types [56].
In a Boolean world consisting of sets, functions and first-order logic the inference $A \Rightarrow B$ as in the classical Venn diagram of Figure 1(a) is the familiar $\neg A \vee B$. Boolean implication in set logic is therefore defined as:

$$\frac{A \Rightarrow B}{\neg A \vee B}$$

It is as if $A \longrightarrow B$ and $C \times A \Rightarrow \neg B$ because there are parts of C outside of B. Two apparent problems remain: first the definition of the concept of negation, second the effect of context. These two problems may really be the same but taking them in turn. In set theory the negation of A is the complement of A and negation is defined by De Morgan's laws:

$$\frac{\neg a \wedge \neg b}{\neg(a \vee b)} \quad \text{and} \quad \frac{\neg a \vee \neg b}{\neg(a \wedge b)}$$

[10] There is a further interesting point arising from this example which is relevant to more complex examples like the law. Δ: 'x is dead' is a sequent formula taking together a sequence of tests as might be carried out by a medical practitioner testing for breathing, pulse, pupil dilation, etc. Again here the conjunction/disjunction of these tests shows the ambiguity of the and/or logical connectives. The category may not be co-cartesian closed [8] unless it is bicartesian closed with finite products and finite co-products ([13].

[11] [56] where Russell first put the paradox in the famous letter to Frege although the paradox itself was first discovered independently by Zermelo [51].

[12] Proof that it is a paradox requires the law of excluded middle [9] at **1** p.1.

Set logic is an idealisation for the real-world. Intuitionistic logic operates only on objects that exist. This is without *tertium non datur*. The corresponding intuitionistic identities are

$$\frac{\neg a \wedge \neg b}{\neg(a \vee b)} \quad \text{and} \quad \frac{\neg a \vee \neg b}{\neg(a \wedge b)}$$

That is the second expression does not hold both ways. The expression $\neg a \vee \neg b$ is not valid (cannot exist) if there is no a and there is no b.
With the principle of *tertium non datur* this is simply:

$$\frac{A}{\neg\neg A}$$

That is if A is true it exists and it is therefore true that $\neg\neg A$ will be true. However just because $\neg\neg A$ is true this does not necessarily imply that A exists.
In the morning twilight world between sets and categories, the real world may be modelled by open or fuzzy sets. The inner dashed circle in Figure 1(b) represents the open set A. If A is open its complement $\neg A$ is closed. The outer circle represents the boundary of the complement $\neg A$. However the complement of a closed set is not necessarily closed [13]. Therefore the $\neg\neg A$, the interior of closure of A may be greater than A ([43], p.53). It is the gap [14] in between which gives rise to defeasance – the natural structure of open systems. The Heyting implication is classically represented by:

$$\frac{C \wedge A \leq B}{C \leq (A \Rightarrow B)}$$

The question of context arises. This can be seen in the Venn diagram in Figure 1(a). There are two contexts over the object A:

1. a local neighbourhood context B; and

2. a global context C – the *universe of discourse*.

B and C may vary but B will always be tied to A. So B is local and C is non-local. Topology is the branch of pure mathematics that can deal with problems of open context. This concept of openness is important for practical reasoning and is at the core of defeasibility. Topology itself has been developed for open sets and

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[13] and therefore more accurately referred to as a pseudocomplement in the Heyting logic below. ([34], at p.221) suggest "we might re-interpret the word 'not' as an operator which generates a new predicate, i.e., by 'not just' we mean 'unjust', where 'just and unjust' is not a logical contradiction ... It is not enough to show that some verbal trick can be used to restore consistency: some reason must be given for thinking that 'not' or the prefix 'un-' does not have its usual meaning in these contexts".

[14] ([34] at p.219) seem even to ascribe this gap in the deontic context to the partial semantic indeterminacy in Plato's theory of ethical language.

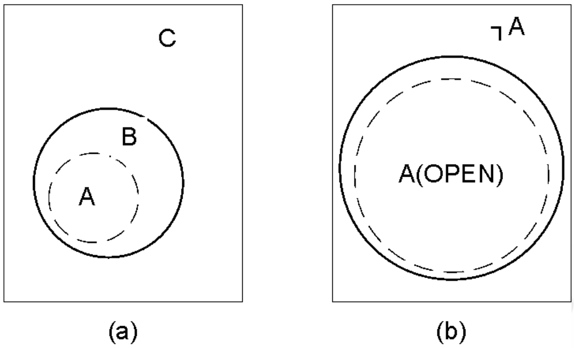

Figure 1: (a) Classical Venn and (b) Open Venn Diagram

therefore aimed at set logic. It therefore has a restricted use in the intuitionistic logic of the real-world. It is generalised in the topos.
Other related instances of openness in pure theory may be found in the way that paraconsistent logic deals with technical explosion ([7] part I) and in the undecidability in intuitionistic logic and modal logic [33]. In applied systems theory the same phenomenon appears in open and free systems [54].

4 Topos Logic

Defeasance arises out of openness. The obvious formal representation of this openness is in open sets which is the basis of the subject of topology. Topological properties are generalised in the concept of a topos. However, rather paradoxically, a topological space with homeomorphisms is not cartesian closed [15]. The cartesian closed is most important in applied category theory because, with limits and exponentials, it has the real world properties of existence and connectivity between objects ([31] at A1.5). The name can be misleading because it is more than cartesian in a cartesian co-ordinate sense and it is more than closed in that it can deal with open properties [25]. This is best presented by the formal arrow in category theory ([44], [6]). A cartesian closed category has finite limits, that is objects which exist (ontologically) and has the (i.e. all possible) connections between them. The concept of limit as a universal was only realised in the last half of the twentieth century [16]. Limits are represented by a pullback diagram of arrows as in Figure 2. The connection between limits and adjointness is discussed

[15] This is proved in ([9], **2** p.353-355). However, there are related special cases like the Grothendieck topos ([9], **3** ch.3) which involves open sets and Hausdorff subcategories of compactly generated spaces and continuous mappings ([9], **2** p.350). Because of continuity and conservation in physics care has to be taken in applying any punctiform [50]. Generally topological spaces are defined on open sets which are sheaves in a topos ([9], **3** ch.9).

[16] Mac Lane asks "Why the discovery of adjoint functors was so delayed?" [44]

in the later paper [54] for open and free systems and for the avoidance of negation [28].
The issues of negation and context and their relationship with inference, lurking in the Venn diagram (Figure 1(a)) need to be made clear. What is the connection between an object A and its context C? Is part of C within A and also within B? These are made explicit and rigorously so in the τ-calculus of a topos [17], a category of categories that are cartesian-closed.

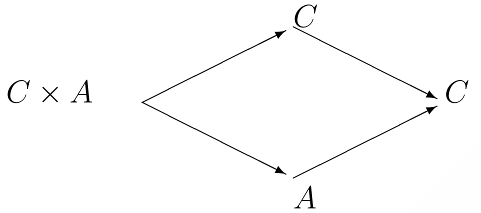

Figure 2: Diagram of Pullback of A over C

A category A [18] in its context C that is context sensitive is represented in Figure 2 by $C \times A$, that is A as determined by its relevant part in C [19]. What about the local context B? This is taken further in Figure 3. The limit is taken over the local context so that B includes the relevant part of C in respect to A together with A itself. This leads to the definition of the Heyting implication, the internal logic of the topos [31], at A1.5.11).

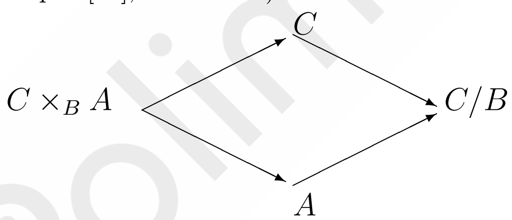

Figure 3: Diagram of Pullback of A over C for that part of A and B in C

Figure 3 is a proof diagram [20] of the definition of B as the conclusion of $A \Rightarrow B$ for a given A in its particular universe C; B is the largest part of C connected to

[17] [31] **2** D4.3 p.963-976.

[18] Because we are concerned with applied categories we are always concerned with large categories not with the category of sets. As there can be no confusion therefore we do not need to resort to more elaborate fonts like the use of bold gothic type in the usual conventions for category theory.

[19] $C \times A$ is an abstraction of multiplication in arithmetic, products in vectors and tensors, intersection in sets, the logical connective AND, etc.

[20] that is a categorial proof up to natural isomorphism by way of composition in a diagram.

A. B is a typing being picked out as a subcategory of C. $A \Rightarrow B$ is the subobject co-equaliser of the two right-hand arrows of $A \longrightarrow C$ and $C \longrightarrow C$. The functor C to B is full and faithful in respect of those objects and arrows that are to be found in B. The subcategory C/B is sometimes known as the slice category and was identified early on as the comma category of particular significance in theoretical computer science [15]. The corresponding intuitionistic inference is given by the pullback over categories.
Both B and A are subcategories of an ambient context category C in the pullback of Figure 3. Here [43] $A \Rightarrow B$ is the largest subcategory of C containing the limit of A with its context. This pullback generalises the Venn diagram above in two respects:

- A is not closed as in the Boolean version (openness).
- The interaction of A with its context C can be anywhere (non-locality) [24].

Openness and non-locality are possibly adjoint. It may well be that these concepts of openness and non-locality are left- and right-adjoint but this needs to be explored further elsewhere.

5 Application

Figure 3 applies to the example that if x is a bird, x flies. The sentence 'x is a bird' identifies an object of the subcategory of the real-world C which is nominated as type 'bird'. A represents a subcategory of C whose objects fly. B is the subcategory of the real-world C that consists of birds that fly. $A \Rightarrow B$ is the inference that x flies. The limit $C \times_B A$ is the concept 'flying birds' which in natural language is a concatenation of particular characters in sequence [23]. Because of the direction of the arrows the pullback has parity and the simple converse may not hold [21]. Then the inference $A \Rightarrow B$ that x flies is only true for that subcategory. C does not precede the inference as required by $C \leq (A \Rightarrow B)$ because C contains objects that can fly that are not birds like an aeroplane.
On the other hand it is easy to see that if C is the moon rather than the real world of the Earth this would radically alter the process of inference and the different possibilities for B because of the different type $C \times A$.
The full analysis of the many possibilities would make C a topos with B a whole collection of possible categories to include for example birds who don't fly because they are very young or because they are moulting at the time or because they are an ostrich. The categories B would be partially ordered in a Heyting lattice as in the constituent internal logic of a topos.
With all the possibilities the form of reasoning is the same even though the outcome may be different. One advantage of category theory is the ability to see

[21] The other way round where C is the subcategory that flies and A the subcategory 'bird' then $C \times_B A$ is 'bird-like fliers'.

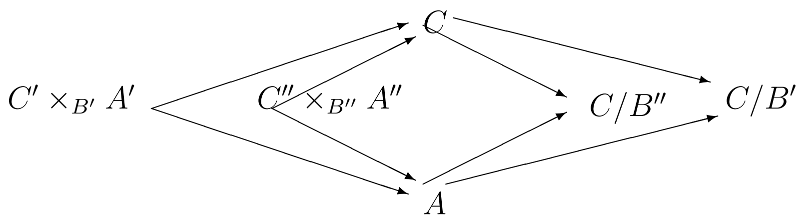

Figure 4: Diagram of Pullback of A over C for B with extensions from B' to B'' and A' to A''

the same structures at different levels (a fractal principle) [22] so that it is possible to abstract with ease and move between levels. The reasoning in Figure 3 can then be abstracted into a more general form corresponding to contextual categories C', C'' with corresponding subcategories A', A'' and slice categories involving B' and B''. Figure 4 shows a pullback made up from limit cones and colimit cocones [6]. Table 1 identifies the symbols in the case of 'x is a bird' and the defeasible reasoning of the dead bird. The last column of the table demonstrates that the same defeasible pullback formalism can be applied to intelligent agents in Java security.

Table 1: Parallel Views of the Universal Pullback of Figure 4

	Implementation	Bird	Dead bird	Java security
C	Universe	bird	bird	Static environment
A	predicate	flies	flies again	class files
B	local type	birds that fly	dead birds that fly when alive	class loader
γ	antecedent	is a bird	is a bird	name space
δ	strengthened antecedent		is dead	untrusted applet
ϕ	inference	x flies	x would fly but dead	security breach

More insight can be gained from the version in Figure 5 of a topos involving the similar arguments and their relationships as a natural composition [14] of pullbacks. The top pullback (the terminal object) provides the abstract version, the two lower are instances but still general. The example for 'x is a bird, dead or an ostrich' are two examples of the instances provided by the interpretations for the categories and arrows in the table. Because of the fractal effect the lower pullbacks in the figure are themselves slice categories with a 'belief revision' arrow (α) between them. Arising out of the principle of adjointness [22] this

[22]For instance an object is an arrow and an arrow is an object.

natural transformation α can be resolved into three components as in Figure 6. Lawvere [38] showed this tripartite relationship with the pullback functor α^* (corresponding to α) both right-adjoint to the existential and left-adjoint to the universal quantifiers: $\exists \dashv \alpha^* \dashv \forall$
Natural defeasance is then possible if these three functors are adjoint as above. The existential functor $\exists$ forces values to exist and the universal quantifier $\forall$ checks for compliance with all the rules in the pullback category on the left in Figure 6. The functor α^* preserves limits between the right- and left-hand slice pullbacks.

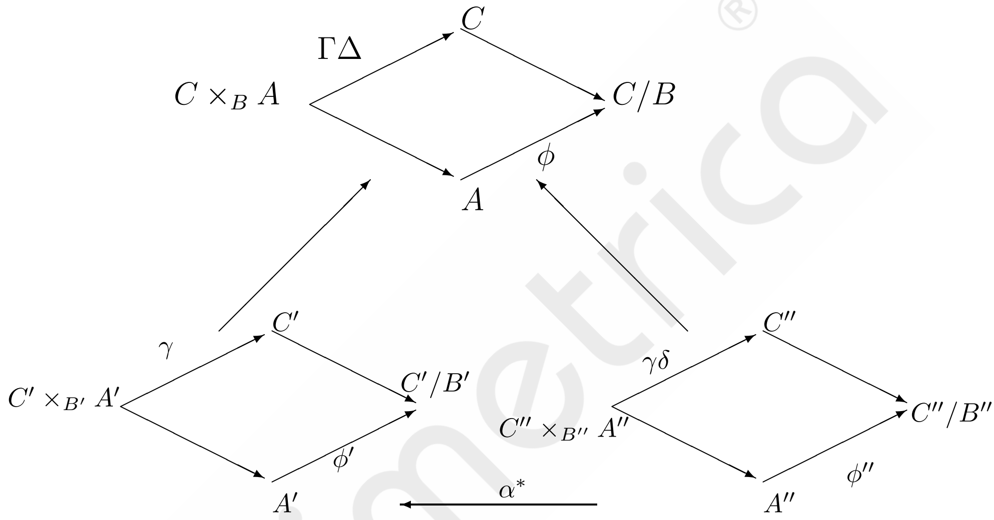

Figure 5: Defeasible Slice Pullbacks

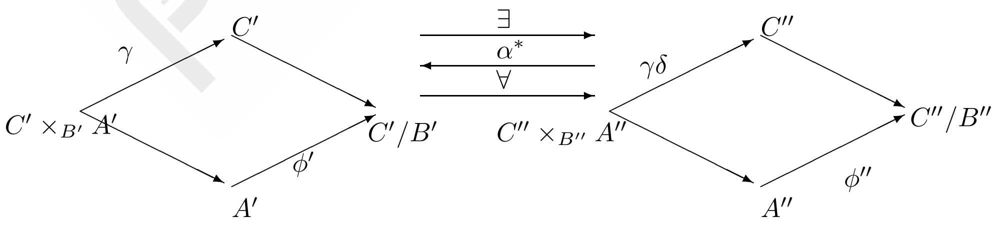

Figure 6: Adjoint functors between Pullback Categories in context of C/B and C/B' respectively

An advantage of going down the categorial road is that natural language [37] and higher order normative concepts can be represented in the same formalism: car licensing [21], legal language [23], computer security [3] and system modelling [27] and indeed classical [24, 39], quantum physics [25], human [32] (but see the comment earlier in the use of the category of sheaves) and machine consciousness [22, 52, 53].
Figures 5 and 6 together mean that terms of logic do not need to be pre-defined and fixed. It is possible therefore as in natural language to have variable intension as well as variable extensions. Figures 6 is also a good example of structures in the sense of Béziau's *research program* and at the same time an implementation of universal logic as *the* type for logical systems. It also exemplifies the applied methodology in the ever continuing battle of sophisticated theory plus idealised application versus simple elegance plus complex example.

6 Conclusions

The Schütte-Ackermann language founded on the axioms of set theory with a calculus based on Boolean logic and used as the reasoning for most formal models of the twentieth century can provide a very useful local snapshot of the real-world but the full non-local picture would need to be built up from an uncountable number of these snapshots. Putting them together gives rise to the phenomenon of defeasance. The full picture is a topos with a Heyting logic where this form of reasoning is natural.
Historically the formal transition in pure mathematics from set theory to the topos has been gradual through open and directed sets, sieves, sheaves, tangent bundles, and manifolds with their associated functions, maps, homomorphisms and étals or more general homeomorphisms [23]. In applied category theory these are more efficiently all elegantly subsumed in the pullback and its dual pushout which can be further abstracted in the Dolittle diagram [26]. Unfortunately details of the pullback await further exposition and a number of applied workers in this field find it still necessary to work in sheaf theory and co-algebra. However, significance of the changing context and of the move from a Boolean to a Heyting world becomes much clearer in the full categorial representation of the pullback. In computer security it is a problem which is currently dealt with by the *ad hoc* application of a 'patch'. The structure of the topos reminds us that a Boolean patch does not give a universal Heyting solution. The real-world does not operate with the logic of an axiomatised Boolean system but with the constructive logic of intuitionistic reasoning.
To sum up, the world as a topos has intuitionistic logic. The internal logic of the topos is Heyting: a entails $\neg\neg a$ but $\neg\neg a$ may not entail a
Every Boolean logic is a Heyting logic but not every Heyting logic is Boolean. The case of defeasible reasoning is an example involving full Heyting logic which

[23] Mac Lane & Moerdijk ch. II [43] discuss these in the context of categories

does not resolve the paradoxes of defeasance but prevents them arising in the first place.

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